Global Management Journal for Academic & Corporate Studies (GMJACS)
Fall 2022, Vol 12 No. 2, PP. 38-51
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Published by Bahria Business School, Bahria University Karachi Campus

Empirical Investigation of Real Exchange Rate between Pak Rupee and US Dollar Employing Markov Switching-AR Model

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Abstract

This study empirically investigates real exchange rate between Pak Rupee and US Dollar employing a two state Markov Switching-AR Model. Bai-Perron test for multiple structural breaks found three structural breaks in the series. Estimation results of Markov Switching-AR model reveal that if the real exchange rate is in state one, its probability of staying in same state in the next period is greater than 99 percent whereas switching to second state is 0.7 percent. Whereas, if real exchange rate is in state two, its probability of staying to the same state is 99 percent and its probability of switching to state one in the next period is less than 0.6 percent.

Keywords: Real Exchange Rate, Markov Switching Model, Structural Breaks, Non-Linear Models

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1. Introduction and Background of the Study

This chapter aims at examining the dynamics of exchange rate between Pak Rupee and US dollar to long run PPP in a nonlinear fashion. Meese & Rogoff (1983) has eloquently documented that linear models of nominal exchange rate fail to predict exchange rate movements. Various other empirical studies, for example, Michael et al. (1997), Sarantis (1999), Sarno (2000) & Baum et al. (2001) have also provided the evidence of the failure of linear models of exchange rate. This resulted in the development of the nonlinear theoretical explanation of the real exchange rate dynamics. Emergence of nonlinear models of exchange rate gave rise to a new set of empirics for explaining and predicting exchange rate dynamics. According to nonlinear theories, one of the important sources of nonlinear movements of real exchange rate is the presence of frictions in the form of transportation costs and tariffs.O'Connell (1998) shows that these sort of frictions result in large deviations from law of one price without participation in arbitrage in goods markets. Sercu, Uppal & Van Hulle (1995) consider transportation cost a key to a relative low adjustment towards purchasing power parity. Theoretical support for presence of cost of transportation as a reason of slow adjustment towards PPP was given by William & Wright (1991), Dumas (1992), Uppal (1993) & Coleman (1995). These studies show that presence of transportation costs may result in nonlinear adjustment of real exchange rate towards PPP. McMillan (2005) attributes nonlinearity in exchange rate dynamics to chartist approach of market participants in formulating exchange rate expectations. Dumas (1992) studied a nonlinear model to observe deviation of real exchange rate from PPP. In this simple model, comprising of only one good and two countries, movement of real exchange rate i.e deviation from PPP should be a mean reverting process, not in its linearity but in nonlinear fashion. This model assumes that markets are spatially separated and transportation costs are assumed to be proportional. He reveals that adjustment is faster if deviation from PPP is larger i.e exchange rate dynamics can only be explained as a nonlinear process. This model also shows that underlying process is divergent inside the transaction band and convergent outside the transaction band. Uncertainty about the persistence of shocks or the presence of sunk cost for arbitrage activities may result in wider transaction band (Dixit, 1989). One of the implications of Dumas' model is; deviation of real exchange rate from PPP may be for a longer period of time, despite not necessarily being a random walk process, as observed by Michael et al. (1997). Various studies consider the real exchange rate process as mean reverting process, probability of its moving away from PPP is greater from moving towards PPP. This asymmetry in movement of real exchange rate is a key reason behind long swings observed in real exchange rate dynamics. Theoretical model suggested by Dumas can be an explanation of the failure in mean reversion of real exchange rate. Bergman and Hanson (2005) show that post Breton Wood real exchange rates among major currencies may be modeled as the two states Markov Switching Regime-AR(1) model.

This study finds that Markov Switching Regime models have better predictability capacity as compared to non-stationary random walk models. The study by Bessec (2000) also employed Markov Switching Regime models and finds that the exchange rates of ERM member countries exhibits mean reversion during credible exchange rate regime, adjusting to PPP during even volatile periods.

A smooth transition autoregressive model was employed by Michael et al (1997). This study reveals STAR model exhibits mean reversion of real exchange rate to PPP in case of deviation. Findings of this study is consistent with the model of Dumas i.e real exchange rate movement is just like a random walk process when deviations from

PPP are small but are mean reverting when deviations are greater. Another such study was conducted by Obstfeld and Taylor (1997) which used band threshold auto regressive (B-TAR) model for capturing nonlinear movement of real exchange rate. In this model, transaction costs are assumed to be the source of nonlinearity in real exchange rate dynamics. In this model, equilibrium value of real exchange rate is not any fixed point, rather it can be anywhere between transaction band. In some of the cases, mean reversion period for real exchange rate is as small as two month when band threshold autoregressive (B-TAR) is employed. Similarly, O'Connell (1998) uses a threshold autoregressive (TAR) model for the estimation of real exchange rate employing data for post Bretton Wood era. But is study, contrary to the study of Obstfld and Taylor (1997), finds that the size of deviation from PPP makes no difference i.e both small and large deviations are equally persistent.

A comparison for the forecasting performance of AR and TAR model for real exchange rate between US/DEM is made by Kuo and Mikkola (2000). This study finds that better forecasting performance of TAR model as compared to that of AR model was not consistent.

This study aims at extending linear models of real exchange rate between Pak Rupee and US dollar to nonlinear models. The studies which employed traditional empirical models reveal that deviation of real exchange rate from PPP last for quite a large period of time. Using these models, it is quite difficult to reject non-stability of real exchange rate if linear models are employed. One of the potential reasons of rejecting such non-stability in real exchange rate is that unit root tests traditionally used have low power if processes involved are near unit root processes. This study is expected to show that there is enough evidence of regime dependency in Pak Rupee/US dollar real exchange rate. This would be expected to unfold stability of real exchange rate series.

The theoretical framework of this study would be Dumas' model because this model provides with the explanation of the failure of mean reversion of real exchange rate. The failure is because of the fact that real exchange rate dynamics is just like a random walk process in case deviations from purchasing power parity are small and it is a mean reverting process when deviation from PPP are quite large. In this study, first, methodology of Markov Switching regime model is employed to investigate the Pak Rupee/US dollar real exchange rate series using data for post Breton Woods's era. Markov Switching Regime model permits the existence of two states in any time series wherein two states shift from state one to state two in a probabilistic way. Hence, shifting from state one to state two occur endogenously and is not imposed by the researcher. That means that in this study, we assume that real exchange rate series is a process which is generated by a stationary Markov Regime Switching two state process. Here one of the states is named appreciation of the real exchange rate whereas the second state is termed as depreciation of the real exchange rate.

1.1 Statement of the Problem

The basic research question this study aims at answering is as follows: "Does the real exchange rate series between Pak Rupee and US Dollar exhibit any sort of structural breaks making it nonlinear? What are the probabilities of regime switching from state one to same state and to state two, if a two state regime Markov Switching (MS) Model is employed?

1.2 Purpose of the Study

One of the specific purposes of this study is to investigate the possibility that deviations of real exchange rate between Pak Rupee and US Dollar from its equilibrium level affects to the short run dynamics in a nonlinear fashion. In this study, we employ two different nonlinear models to resolve the problem of slow real exchange rate convergence to its long rum equilibrium level.

Specifically, this study investigates whether Markov Switching (MS) can give any additional insight into real exchange rate dynamics between Pak Rupee and US Dollar? The theoretical basis of our study is Dumas (1992), Uppal (1992), Sercu et al (1995) and Obstfeld and Rogoff (2000). All of these studies show that adjustment of real exchange rate towards the purchasing power parity when market frictions are present out there, there is necessarily a nonlinear process.

2.Literature Review

Last two decades have witnessed an ever increasing interest among econometricians in specifying, testing and estimating of models in a nonlinear fashion (Potter 1999). The data generating mechanism behind macroeconomic time series are better understood by using nonlinear specification for estimation purpose. Potter suggests that at least two requirements must be satisfied for properly modeling macroeconomic time series in nonlinear fashion so that nonlinearities in the concerned macroeconomic time series may be properly captured.

The first condition to be fulfilled that time series under investigation actually follow a nonlinear process. The second required condition is that we use a statistical method which reliably capture all the possible nonlinearities present in the series we are investigating. First condition is not difficult to be fulfilled. However, to fulfill the second condition is a bit problematic.

2.1 Purchasing Power Parity and Nonlinear Adjustment

The purchasing power parity in its long run version can be written as follows:

$$St = \beta_0 + \beta_1 p_t + \beta_2 P_t^* + q_t \qquad (1)$$

In this equation, St is used for the log of nominal exchange rate (we measure nominal exchange rate as price of foreign currency in term of domestic currency). pt indicates the log of domestic price index whereas Pt* denotes the log of foreign price index. qt represents real exchange rate and is used in the above equation as the error term. Under the null hypothesis, error term follows a stationary process. Purchasing power parity, in its strong form, implies a joint restrictions of symmetry i.e $\beta 1 = \beta 2$ and proportionality i.e $\beta 1 = - \beta 2 = 1$. But empirical data do not verify these restrictions. This inconsistency is due to measurement error (Taylor 1988 and Cheung and Lai 1993). Other reasons of this inconsistency are differential composition of general price indices (Patel 1990) and differential shocks in productivity (Fisher and Park 1991). This has been also pointed out by Baum et al. (2001). However, weak form of purchasing power parity imposes a necessary condition that real exchange rate qt is a stationary process in the long run without any sort of parametric restrictions.

One of the useful ways of testing purchasing power parity theory in its long run form is linear form cointegration technique. Although cointegration methodology presumes a behavior of mean reversion type of relationship among the variables under consideration, conventional linear form cointegration methodology presumes a linear process. Linearity of the process implies that adjustment towards the equilibrium is not only continuous but has constant speed of adjustment toward the equilibrium. However, the possibility of presence of nonlinearity in the system in equation 1 may be one of the important causes of failure if convention form of nonlinear cointegration technique is employed. Pippenger and Goering (1993) suggested that presence of transaction boundaries or possibility of nonlinear dynamics of real exchange rate series shows the usefulness of the linear cointegration technique and conventional unit root testing. Their study finds that power of the traditional unit root test and cointegration methodology drastically falls under the threshold process. In addition to this, there is an important line of research which considers the value of nonlinearity in case of error correction technique of series of real exchange rate moving towards its long run equilibrium level. Such type of error correction mechanism having nonlinearity may not be captured by nonstationarity test of linear form (Sarno 2000).

There are several possible explanations of nonlinearities in the movement of real exchange rate series, as explanation offered by economic theories. Lucas critique suggests that a shift in economic policy usually causes breaks in trends of the most of the macroeconomic series. These breaks result in relatively larger swings in the series. Conventional random walk models do not capture the effects of even obviously observed changes in economic policies. For example, a study by Kaminsky (1993) theoretically reveals that changing contractionary monetary policy into expansionary monetary policy enhances the extinct of depreciation in exchange rate. Assuming the flexibility in prices are not same as the flexibility of exchange rate, same reasoning may be applicable to the real exchange rate series. Moreover, one of the important sources of the nonlinearity observed in real exchange rate series is said to be the heterogeneity of the participants trading in the foreign exchange market, as suggested by study of Sarantis (1999). A study by Peters (1994) and another study by Guillaume et al (1995) shows that heterogeneity in foreign exchange market is due to different horizon for investment, different geographic locations, differences in risk profiles and various type of institutional constraints.

A study by Dumas (1992) argues that adjustment of the real exchange rate towards its purchasing power parity equilibrium level is nonlinear in nature and not the linear one. According to this study, the failure of law of one price to hold is mainly because of the presence of the transaction costs. In the presence of transaction costs Ci, the price of good i in location A (PiA) may not be same as in location B (PiB). In such case, the relative price would fluctuate in the range of $1/Ci \leq PiA / PiB \leq Ci$. The nonlinearity comes into this case because presence of transaction costs, Ci , renders an arbitrage non-profitable in case of a small deviation from the law of one price. In such situation mean reversion in real exchange would not be there because of small deviation from law of price. However, sufficiently large deviation from law of one price makes arbitrage profitable. In such case, mean reversion may be observed.

As a host of macroeconomic time series has exhibited nonlinearities in their behavior, various nonlinear models have been developed. Markov Switching models are of great significance. These models have also been employed for the investigation of the nonlinear macroeconomic time series pertaining the economy of Pakistan. One such study is by Safia Minhaj and Mohammad Nishat (2018). This study employed Markov Switching Model for the presence or otherwise of exchange rate pass through. It reveals an association between exchange rate pass and inflation targeting. Findings of this study show that regime switching was not only sudden but sporadic too till 1990. Tayyab Raza Farz et al (2020) compared the forecasting performance of nonlinear models with that of linear models for the purpose of evaluating forecast outlook of international organizations like IMF and OECD.

This study employed SETAR model and Markov Switching-AR model for accounting for nonlinearity, whereas, linear model estimated are AR and ARMA models. Macroeconomic variables estimated for forecasting purpose are GDP growth, CPI and exchange rate for G-7 countries. Results of this study show that the forecasting performance of nonlinear models is better than that of linear models.

3. Methodology

3.1 Markov Switching Model

Markov Switching Regime models differently vary from the models wherein the regime change or the breaks in trend is imposed exogenously. Whereas, in Markov Switching models regime switching is stochastically determined. In these models inferences are made on the basis of probabilistic outcomes which are most likely to prevail on any point of the observed time series. This is considered to be one of the major advantages of the Markov Switching models wherein flexibility of modeling regime switching is stochastic in nature. In a two state Markov Regime Switching can very conveniently be employed to study the movement of a real exchange rate series. An appreciation of real exchange rate may be considered regime one whereas depreciation of real exchange rate may be taken as regime two in case of employing a two state Markov Regime Switching model. A random process is considered to be governing switching between these two states or regimes (Klaassen, 1999). An important study by Engel & Hamiton (1990) employed Markov Switching model with drift to investigate the long swings in the exchange rate series. They suggested that Markov Switching models are far better than the random walk model for studying the dynamics of exchange rate series. Their study reveals that the mean squared error was far lower when Markov Switching model is employed than the mean squared if a single regime random walk model is employed. Engel & Kim (1999) investigate the nature of the behavior of UK/US real exchange rate based on the assumption real exchange rate series between these two currencies is I(1) i.e is integrated of order one. This study suggests that any deviation from any permanent value may be modeled as Markov Switching Regime having the states. In their model, these three states are described as low variance, medium variance and high variance.

One of the characteristics of Markov Regime Switching process is that the probability of being the underlying process in a particular state depends on the state the process was in previously. This model was originally employed by Hamilton’s (1989 and 1990) studies on dynamics of exchange rate series. Our study employs the model used by Bergman and Hanson (2005). This model estimates the size of the autoregressive parameter. Contrary to this, Hamilton’s study presumed it to be unity in his exchange rate model. Our study uses only one autoregressive term. The reason for this is that autocorrelation in exchange rate in short term is generally consider to be very small. This proposition is in accordance with the argument of West & Cho (1995). However, a restriction of the ergodicity about the regime process is assumed. Weak form stationarity is also assumed for the real exchange rate series. Our study presumes that the probability of switching from a regime to another one is invariable over time.

The model of this study comprises of two components. First component of the model is regime process comprising of two regime paths, s1 and s2, which are unobservable. Whereas, second element of the model, is the equation of the mean. The mean real exchange rate changes between these two regimes are µ1 and µ2. These parameters are assumed to be invariable over time. In case of different mean regimes, persistence of different mean regimes causes long swings in the series under investigation. Bergman and Hanson (2005), employed following model of real exchange rate for estimating Markov Regime Switching model;

$q_t = \mu_{st} + \alpha_{st} q_{t-1} + \varepsilon_t$ (2)

where q_t is the real exchange rate, ε_t is N(0, σ_t^2) i.e error is assumed to be normally distributed with mean 0 and variance σ_t^2. Initial value of real exchange rate is assumed to be fixed at q_0. In the above equation, changes in the natural log of real exchange rate are assumed to be normally distributed having the mean µi and variance w σ_i^2 here si = 1 and 2 showing two possible states i.e two possible regimes. When s_i = 1, the change in real exchange rate is assumed to be μ_1 and when s_i = 2, change in real exchange rate would be μ_2. Unobservable random variable si, indicating two different states or to different regimes, is independent of the past level of qt conditional upon st-1. Depreciation in real exchange rate and appreciation in real exchange rate are modeled as regime switching process (stochastic process) which generate growth rate of series of real exchange rate. These regimes can be associated with conditional distributions of real exchange rate's growth rate. In these distributions, the mean of the series is assumed as positive in first regime whereas, it is assumed to negative in the second one. The regime path (st-1, st-2,…….) is assumed to follow a Markov Process of order first having transition probabilities of homogenous time.

$P(st = j \mid st\text{-}1 = i) = pij$ (3)

Where, i and j are 1,2. This Markov process can be summarized by the transition probabilities as follows;

$P(s_t = 1 \mid s_{t-1} = 1) = p_{11}$

$P(s_t = 2 \mid s_{t-2} =1) = p_{21} = 1- p_{11}$

$P(s_t = 2 \mid s_{t-1} = 2) = p_{22}$

$P(s_t = 1 \mid s_{t-1} = 2) = 1- p_{22}$ (4)

Here p_{ij} is the probability of this Markov chain model switching from the state I in time period $t-1$ to the state j in the next time i.e in time period t. This regime path process is dependent on realized past value of the under study series of real exchange rate, qt. Whereas, the regime path i.e s_t is dependent on its previous value i.e on s_{t-1}. Here it is important to notice that real exchange rate is not assumed to experience long swinging. However, regime persistence may exhibit asymmetry. The means of the regimes μ_1 and μ_2 carry opposite signs. The probabilities of staying in both the regimes, p_{11} and p_{22} would be relatively large in case of long swinging.

According to Hamilton (1994), unconditional probability of the process being in each regime may be derived by following equations;

$P(s_t = 1) = 1 – p_{22} / 2 – p_{11} – p_{22}$ (5)

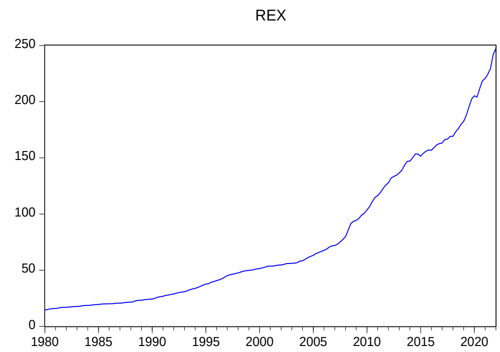
$P(s_t = 2)\ 1 – p_{11} / 2 – p_{11} – p_{22}$

These equations are imperative for the Markov Switching models, for these are directly concerned with long swing cases. For estimating the parameters and the transition probabilities which govern the Markov chain in case of unobserved state, a reiterative estimation technique is required. For each possible state, the likelihood is estimated. The probability of any specific state to occur is calculated by division of the likelihood of specific state by sum total of the likelihood of both the states. Expectations maximization algorithm has been used by Hamilton (1990). This algorithm is useful in the case where all the parameters vary. Expectations maximization algorithm proposed by Dempster et al., (1977) has been introduced for models where observed series is dependent on any unobservable process. For making inference from Markov chain models, smoothing of the estimates is required, so that information contained in the whole sample may be utilized. After this smoothing estimates of the probabilities are interpreted in such a way that probability of occurring a specific state is greater than 0.5.

3.2 Description of Data

For the purpose of estimating a two regime Markov Switching model, we employ quarterly data for real exchange rate between Pak Rupee and US Dollar, ranging from 1980Q1 to 2022Q1. The data has been obtained from IFS.

Figure 1: REX trend



4. Estimation of a Two Regime Markov Switching Model

4.1 ADF Unit Root Test

Real exchange rate series, in log form, between Pak Rupee and US Dollar exhibit a highly nonlinear as shown below. The graph of the series under investigation exhibits not only trend indicative of non-stationarity but regimes switching too. Hence, we first conduct a test for stationarity employ ADF unit root test. The results of ADF unit root test at level and first difference of the series are shown in table1. The ADF unit root results reveal that the real exchange rate series is not only non-stationary at level, it is non-stationary when ADF is employed for first difference form of the series. However, ADF test results in this table reveals that series under investigation is stationary at 2nd difference.

4.2 Breakpoint Unit Root Test

Perron (1989) has found that if the time series under investigation has any sort of structural break, traditional unit root tests, such as ADF test suggested Dickey and Fuller (1979 & 1981), give biased results towards non rejection. Hence, our study also tests for unit root with breakpoint. The results reveal that our series is stationary at 2nd difference if even if we employ breakpoint unit root test.

4.3 Bai-Perron Test for Structural Break

On account of the graphical representation of our series of real exchange rate, results of ADF test and Breakpoint unit root test, we expect structural breaks in our series. Besides, frequent switches in monetary policy stance and exchange rate regime also make us believe the presence of structural breaks in our series of real exchange rate. Hence, we employ Bai-Perron test for structural break. Results are shown in tables 2 and 3.The results in Table 3, reveal that, in our series of real exchange rate, there are 3 structural breaks, one n 1st quarter of 1997, 2nd in 4th quarter of 2003 and 3rd in 1st quarter of 2010. It shows that real exchange rate series under investigation exhibits regime switches, hence, we estimate a Morkov Switching-AR Model for real exchange rate between Pak Rupee and US Dollar.

4.4 Estimation of Markov Switching Model

Markov Switching Model, original employed by Himilton (1989 & 1990), differs from other models in that other models impose exogenously determined breaks, whereas Markov Switching Models impose the breaks in such a way that their timings are stochastic in nature.

According to Klassen (1999), the idea behind a two state Markov Switching Model for a series of real exchange that it divides the real exchange rate in two regimes, an appreciation regime and a depreciation regime. Hence, we estimate a two state Markov Switching AR model. In this study, we estimate as employed by Bergman and Hanson (2005) which estimates the size of the AR parameters, contrary to that Himilton's model which employed AR with only one lag. Model estimated is as follows:

$$q_{t} = \mu_{st} + \alpha_{st} q_{t-1} + \mathcal{E}t \qquad (4.1)$$

Table 1: ADF Results

ADF at Level		t-Statistic	Prob.*
		3.036	
Test critical values:	5% level	-2.879	1
ADF at 1st Difference		0.507	
Test critical values:	5% level	-2.879	0.987
ADF at 2nd Difference		-7.777	
Test critical values:	5% level	-2.879	0

Table 2: Bai-Perron Test

Sequential F-statistic determined breaks:		3	
		Scaled	Critical
Break Test	F-statistic	F-statistic	Value**
0 vs. 1 *	708.1438	708.1438	8.58
1 vs. 2 *	88.65016	88.65016	10.13
2 vs. 3 *	16.09881	16.09881	11.14
3 vs. 4	7.532874	7.532874	11.83

* Significant at the 0.05 level.

** Bai-Perron (Econometric Journal, 2003) critical values.

Table 3: Structural Breaks

	Sequential	Repartition
1	2010Q1	1994Q3
2	1997Q1	2003Q4
3	2003Q4	2012Q2

5. Results and Empirical Analysis

In the estimation of Markov Switching Model for real exchange rate series, we employed AR(4) i.e. 4 lags for AR term. The estimated probabilities of switching from one state to another are given below in Table 4. The results reveal that the probability of switching to regime one, if series is already in state one is 0.992405 and two state two is 0.007595. If the real exchange rate is currently in state two, the probability of switching it to state one is 0.005707, whereas that of staying in the state 0.994293. One-state ahead predicted regime probabilities are shown in the figure 2. The constant expected duration of the real exchange rate in two different regimes are given below in table 5. This reveals that the expected duration of real exchange rate staying in state one is 131.6619 quarters whereas, expected duration of our series staying in state two is 175.2177 quarters.

Table 4: Markov Switching Model for real exchange rate series

	1	2
1	0.992405	0.007595
2	0.005707	0.994293

Table 5: Constant expected duration

State 1	State 2
131.6619	175.2177

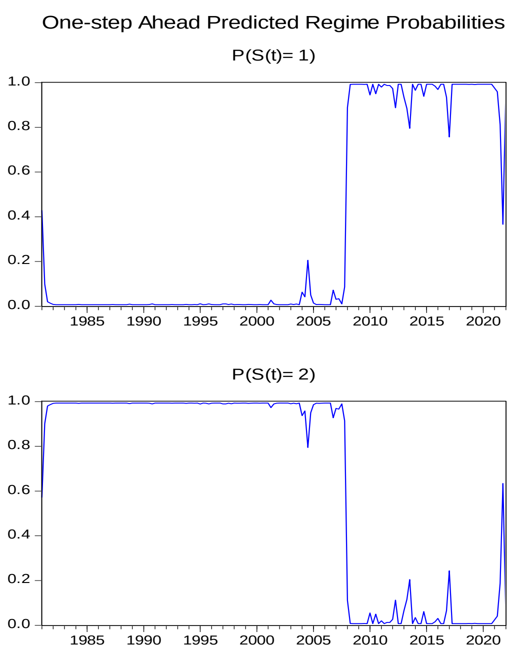
6. Conclusion and Recommendations

The results of this study reveal that real exchange rate series between Pak Rupee and US Dollar is not only non-stationary at level but at 1st difference too when tested when DF test for stationarity is employed. Same results are found when series is tested for stationarity employing breakpoint ADF test. Bai-Perron test for multiple structural breaks has also be conducted, wherein three structural breaks are found in the series of real exchange rate, 1st in the 1997Q1, 2nd in 2003Q4 and 3rd in 2010Q1. The estimation results of a two regime Markov Switching-AR model with 4 lags of AR term reveal that probability of switching from state one to the same state is greater than 99 percent and to state two is 0.7 percent.

Similarly, the probability of switching from state two to the same state in the next period is also more than 99 percent, whereas the probability from switching from state two to state one is less than 0.6 percent. Moreover, for the purpose of further empirical investigation of real exchange rate, Self-Exciting- TAR Models and smooth transition autoregressive (STAR) models should also the estimated.

Note: *This paper is a part of the Scholar's PhD dissertation*.

Figure 2



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